c = constant
f = deviation in stream function from the undisturbed laminar profile
$f(r)$ = function describing wave shape of disturbance in Equation (10)
g = deviation in vorticity from the undisturbed laminar profile
h = radius of conduit or half the plate spacing
N_{Re} = Reynolds number $U^o h/\nu$
r = dimensionless radial coordinate
t = dimensionless time
u, v = dimensionless velocity components in x, y directions, respectively
u' = deviation in u from undisturbed laminar profile
U^o = maximum (center line) velocity of the steady laminar flow
x = dimensionless coordinate downstream from disturbance
β = dimensionless disturbance frequency as defined by Equation (10)
ψ = dimensionless stream function
ω = dimensionless vorticity
ν = kinematic viscosity

LITERATURE CITED

1. Aziz, K., and J. D. Hellums, *Phys. Fluids*, **10**, 314 (1967).
2. Birkhoff, G., "Hydrodynamics," Princeton Univ. Press, N. J. (1960).
3. Chandrasekhar, S., "Hydrodynamic and Hydromagnetic Stability," Clarendon Press, Oxford (1961).
4. Corcos, G. M., and J. R. Sellars, *J. Fluid Mech.*, **5**, 97 (1959).
5. DeSanto, D. F., and H. B. Keller, *J. Soc. Ind. Appl. Math.*, **10**, 569 (1962).
6. Dixon, T. N., Ph.D. thesis, Rice Univ., Houston, Tex. (1966).
7. Eckhaus, W., "Studies in Non-Linear Stability Theory," Springer-Verlag, New York (1965).
8. Ekman, V. W., in "Modern Developments in Fluid Dynamics," p. 321, Dover, New York (1965).
9. Fromm, J. E., and F. H. Harlow, *Phys. Fluids*, **6**, 975 (1963).
10. Gill, A. E., *J. Fluid Mech.*, **21**, 145 (1965).
11. Leite, R., *ibid.*, **5**, 81 (1959).
12. Lin, C. C., "The Theory of Hydrodynamic Stability," Cambridge Univ. Press (1955).
13. Meksyn, A., and J. T. Stuart, *Proc. Roy. Soc. (London)*, **A208**, 517 (1951).
14. Sexl, T., *Ann. Phys. (Leipzig)*, **83**, 835 (1927); **84**, 807 (1928).
15. Shen, S. F., "Theory of Laminar Flows," Vol. IV, Princeton Univ. Press, N. J. (1964).
16. Squire, H. B., *Proc. Roy. Soc. (London)*, **A142**, 621-628 (1933).
17. Watson, J., *J. Fluid Mech.*, **14**, 211 (1962).

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Some Observations on the Effect of Interfacial Vibration on Saturated Boiling Heat Transfer

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An experimental investigation was carried out to determine the effects of vibration of the heat transfer surface in saturated pool boiling of water at atmospheric pressure. Wires of 0.01 in. diameter were heated electrically and vibrated electromagnetically at frequencies ranging from 20 to 115 cycles/sec. and amplitudes from 0.0118 to 0.0701 in. An increase in heat transfer up to a maximum of 200% at low ΔT was observed for an increase in frequency and/or amplitude. At a heat flux of 10^5 B.t.u./(hr.)(sq. ft.) high-speed motion pictures were taken at 4,800 frames/sec. of a wire vibrating at 45 cycles/sec. with an amplitude of 0.0492 in. Comparison of these films with those taken at the same heat flux without vibration showed that the generating period and diameters at break-off for the pulsed wire follow normal distribution. The waiting period is much longer and more fluctuating in nature. A slight increase in bubble emission frequency was also observed for pulsating wire.

Experimental and theoretical investigations in the field of nucleate boiling heat transfer have indicated that the flow of heat is partly from the surface to the liquid through the boundary layer and partly by latent heat transported through the vapor bubbles. However, photographic studies on subcooled boiling by Gunther and Kreith (*1*), Rosenhow and Clark (*2*), and others have proved that heat transfer takes place primarily through the boundary layer, the transport by latent heat being negligible. As the large heat fluxes obtained cannot be explained on the basis of thermal conduction through the superheated layer, it has been postulated that certain random microconvection is created in the boundary layer by bubble dynamics, reducing thus the high thermal resist-

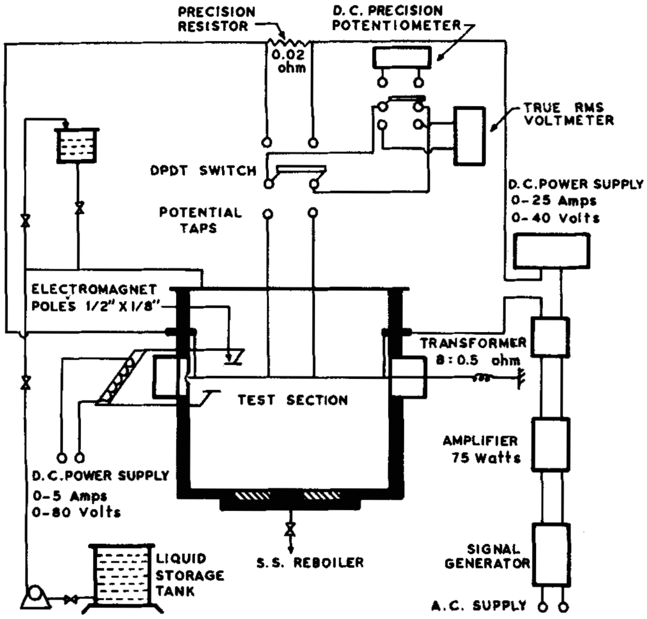

Fig. 1. Details of the experimental set-up.

ance of the boundary layer considerably.

This explanation however is not valid over the complete range of saturated nucleate boiling. In the discrete bubble region, that is, for small temperature differences, the heat transfer is largely through the boundary layer. Both the bubble population and bubble emission frequency increase considerably with increasing temperature difference. When the heat flux reaches around 80% of its critical value, it is likely that the latent heat transport becomes dominant over other modes of heat flow.

Based on the above findings it is reasonable to assume that any external agitation in the thermal boundary layer or any device to increase the bubble population or bubble emission frequency would enhance the heat flux in subcooled boiling and in the lower range of saturated boiling. Westwater (3), in his microscopic studies of bubble growth, found that the heat transfer coefficient for a bubble decreases as the bubble grows; the typical variation of heat transfer coefficient h with time being

θ, sec.	h, B.t.u./(hr.)(sq.ft.)(°F.)
10^{-4}	2,400 to 8,400
10^{-3}	379 to 1,900
10^{-2}	57 to 420

He concluded that any mechanism to remove bubbles from the surface when they are young would increase the heat flux. Schweppe and Foust (4), Robinson and Katz (5), and others showed that agitation of the liquid increased the heat flux in the lower nucleate boiling region but had little effect near the peak heat flux. In all previous work the agitation was imparted to the bulk of the fluid. The effect of turbulence in the boundary layer alone was not considered. Furthermore, no extensive study of the behavior of bubbles under vibrational conditions has been undertaken.

The object of the present investigation was to study the statistical behavior of bubbles growing on a pulsating heat transfer surface and to present the heat flux data under certain conditions of vibration. Vibrations of uniform amplitude were imposed on the surface by an electromagnetic device. This eliminated to a large extent the effect of any external agitation in the fluid system except in the boundary layer.

APPARATUS

The boiling apparatus consisted of a rectangular stainless steel vessel provided with a vertical ¼-in. thick Pyrex glass window for visual observation and high-speed photographic work. The inside dimensions of the vessel were 9 in. by 6 in. by 11 in. high. At the level of the heat transfer surface, there are two pockets (each ½ in. by ⅛ in. by 1 in long) on opposite walls through which the poles (½ in. by ⅛ in. in cross section) of the d.c. electromagnet are inserted (see Figure 1). The gap between the pole pieces, when fitted in position, is ½ in. The d.c. electromagnets could produce a magnetic field with an intensity of 10 K gauss. The input to the electromagnet was fed from a custom-made regulated d.c. power supply (0 to 80 v., 0 to 5 amp.). Four electric strip heaters, each with a capacity of 250 w., were welded to the bottom and sides of the vessel to keep the liquid at its saturation temperature under atmospheric conditions. The system was insulated with 1-in. thick asbestos except for the glass window.

The heat transfer surface was chemically pure-grade platinum wire of 0.01 in. diameter. The wire also served as a resistance thermometer. To eliminate end effects and to have uniform amplitude of vibration, only a small central part of the wire was used as a test section.

The heating element was stretched across the vessel through two Teflon supports which were screwed to the opposite sides of the vessel. One end of the wire was fixed on the rear end of a Teflon support, while the other end was connected to a compression spring through the second piece of Teflon support. A silicone diaphragm was used to seal the leak on this side. Potential leads of 0.0045 in. diameter were fused with the heat transfer surface to measure potential drops across the test section.

The electrical circuit diagram for this set-up is shown in Figure 1. An a.c. signal of varying frequency supplied by an audio signal generator was amplified through a 75-w amplifier. A special transformer was designed which matched the impedance of the amplifier and of the external circuit. This helped in operating the amplifier at a higher efficiency. The a.c. signal was superimposed with d.c. through a regulated d.c. power supply (0 to 40 v., 0 to 25 amp.) which was connected in series with the a.c. circuit. A precision resistor of 0.02 ohm was also connected in series for measuring d.c. current in the circuit. A precision potentiometer, capable of reading up to 0.05 mv. was used to measure d.c. potential drops, while a true root-mean-square voltmeter, capable of reading up to 0.1 mv. was used to measure a.c. potential drops. The amplitude of vibration along the test section was measured by a precision cathetometer and the frequency of wire was measured by a stroboscope.

EXPERIMENTAL PROCEDURE

The platinum potential leads and heating leads were fused to the main heat transfer surface and then annealed for 15 min. with a.c. current. The test section of the wire was cali-

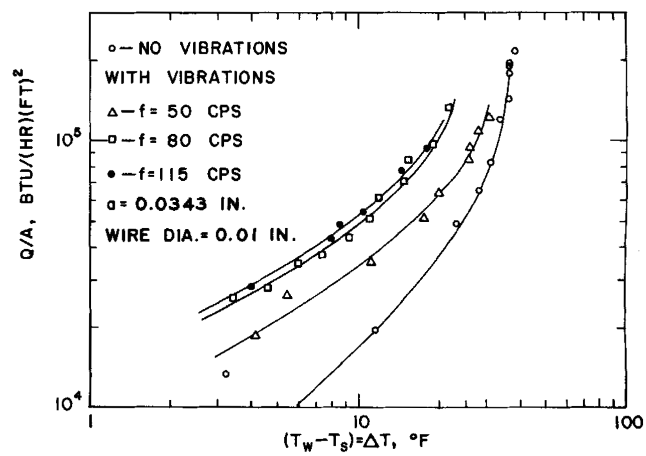

Fig. 2. Influence of vibrations on natural convective and nucleate boiling heat transfer.

brated for its resistance at 32° and 212°F. The calibration was performed by passing a very low d.c. current and measuring the potential drops across the precision resistor (0.02 ohm) and the test section. At least ten sets of readings were taken for each temperature, and average values were used for calculation purposes.

Since at a given value of the frequency and tension on the wire, the amplitude of vibration of the wire is mainly controlled by a.c. current in the wire and d.c. current in the electromagnet, it was desirable to hold these two values constant for the entire run of the experiment. However, even with this control, the amplitude varied as the temperature difference was increased, most likely due to the interference of d.c. current with the original magnetic field produced by the a.c. current. Therefore, to maintain constant amplitude of vibration, the adjustment of the d.c. magnet current was necessary from time to time.

For the measurement of the heat flux and temperature difference, the d.c. current was increased in steps and the d.c. potential drops across the test section and the precision resistor, as well as the a.c. potential drop across the test section, were measured as mentioned above. For calculating temperature differences the relations for platinum resistance thermometry (*12*) were used.

To study the growth rates of bubbles, a high-speed 16-mm. Fastax camera, operating at 4,800 frames/sec., was used. The vapor columns originating from the natural active sites were photographed for both vibrating and stationary wires at the same heat flux density. The films were analyzed frame by frame in an editing machine.

HEAT FLUX

Measurement of heat flux density and corresponding values of the temperature driving force were made in the entire region of nucleate boiling for various frequencies and amplitudes. The observed experimental results are shown in Figures 2 and 3. The heat flux to the boiling liquid increased with the increasing vibrational conditions. This trend holds throughout the nucleate boiling region. Below a temperature difference of 10°F. no bubbles were formed and hence the heat transfer was due to natural convection only. The enhancement in heat flux obtained in this region confirms the effect of vibrations on natural convective heat transfer established by Lemlich (*6*) and many other investigators. All the curves plotted have a tendency to converge as the critical heat flux is approached.

Figure 2 shows the results for a constant amplitude of 0.0342 in. and several frequencies between 50 and 115 cycles/sec. Additional experiments were conducted in which the frequency was fixed and the amplitude was varied. The results of these experiments are shown in Figure 3. Figure 4 shows the effect of average wire velocity on heat flux density at various temperature differences. The increase in wire velocity increases heat flux at all temperature differences. At a given velocity the vertical distance between the heat flux curves continues to decrease, indicating that the effect is more pronounced at low temperature differences. In this particular plot it can be seen that the heat flux data around average wire velocity of 0.2 ft./sec. are widely scattered. However, no definite explanation of this unsteady phenomenon could be given at this stage. A more detailed study is needed to determine the reason for such behavior. The observation of the thermal boundary-layer behavior, by the Schlieren technique, during vibration of the heat transfer surface, is currently under way.

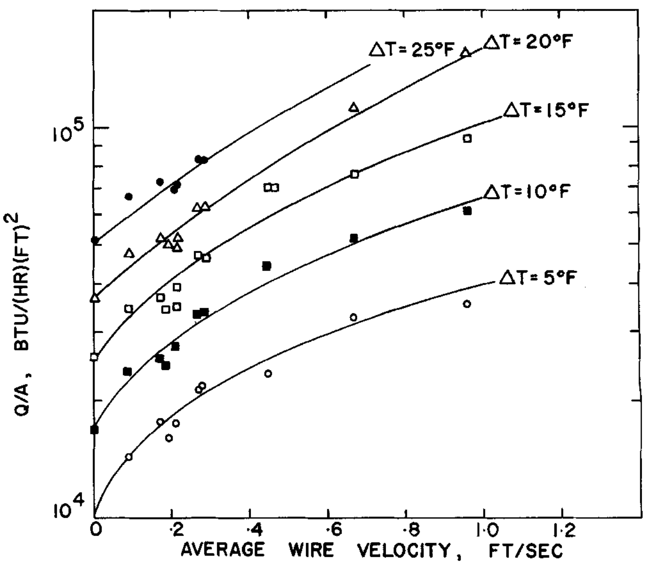

Fig. 4. Influence of vibrations on natural convective and nucleate boiling heat transfer.

STATISTICAL BEHAVIOR OF BUBBLES

The evolution of a bubble is believed to take place in three distinct stages. The first stage is marked by the development of the boundary layer when a superheated liquid layer is formed and a bubble starts growing on a potential active site. In the second stage the boundary layer is disturbed considerably by the further growth of the bubble. The interfering action of a bubble extends to as far as one bubble diameter away from the nucleation site

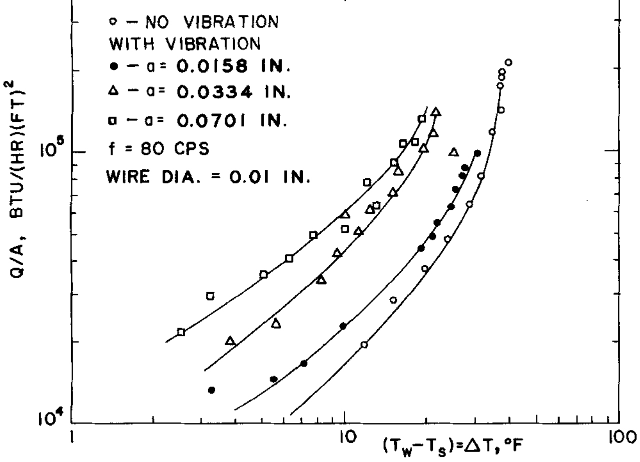

Fig. 3. Influence of vibrations on natural convective and nucleate boiling heat transfer.

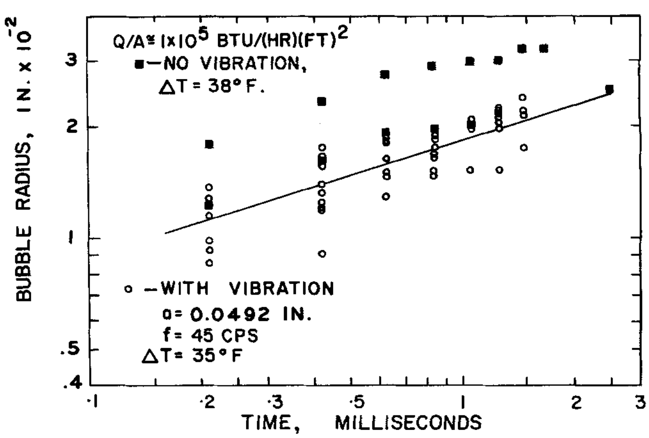

Fig. 5. Bubble growth rate.

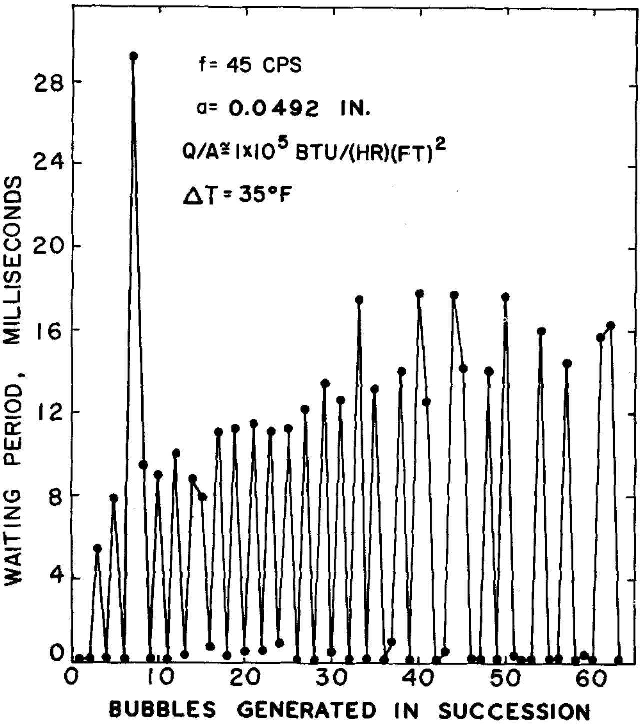

Fig. 6. Fluctuations in waiting period.

(7). The third stage can be characterized by the destruction of the boundary layer. After the bubble leaves the surface, relatively cold liquid flows toward the nucleation site and a finite time is required for the cooler liquid to warm up to the proper superheat before the cycle repeats itself. In studying the mechanisms of boiling heat transfer, the statistical bubble analysis is of great assistance, since the bubble behavior involved is inherently random.

In the present investigation, experiments* were conducted to study the various stages in the evolution of a bubble under vibration conditions with high-speed photography. A film, taken at 4,800 frames/sec. and a heat flux of approximately 10^5 B.t.u./(hr.)(sq.ft.), was analyzed frame by frame. The vibrational conditions of the wire during the experiment were: frequency = 45 cycles/sec., amplitude = 0.0492 in., a temperature difference of around 35°F. Results of the above experiment were also compared with another film taken at the same heat flux but without vibrations. The temperature difference required in the latter case was 38°F., 3°F. higher than the first one.

BUBBLE GROWTH RATE

Bubble growth takes place as a result of evaporation at the bubble wall, the heat for this evaporation being transferred by conduction from the surrounding fluid. The bubble growth rate is thus governed by the temperature distribution in the boundary layer as well as its thickness. Since these two factors would be affected by the onset of vibrations, it may therefore be expected that the bubble growth rate would be different from the one without vibration. In the present investigation it was not feasible to determine the actual localized values for the temperature distribution and thickness of the thermal boundary layer, and hence no theoretical model was attempted. However, an equation similar to the one proposed for stationary surfaces has been evaluated.

The equations proposed to date for stationary surfaces (*8, 9*) have the following form:

$$r = c\,\theta^n$$

To develop a corresponding equation for a pulsating heat transfer surface, the film was analyzed frame by frame with a magnification of 1:44. About 50 values of r and θ were read from the film for various bubbles growing on one particular natural active site. The best fit line was drawn for the above data by the least square method and the values of n and c are thus evaluated as

$$n = 0.3154$$

$$c = 0.1612$$

A comparison of the growth rate of bubbles under conditions of pulsating and stationary wires was then made. In both cases the heat flux density was maintained at the same value. Since the surface temperature was higher in the case of a stationary wire, the number of active sites was also greater. The values of r and θ, determined from the film for stationary and pulsating surfaces, are plotted in Figure 5. As is evident from Figure 5 three distinct stages of growth rate of bubbles exist for stationary surfaces. During the initial stage of growth ($\theta = 0$ to 0.2×10^{-3} sec.), the rate is the fastest of all the stages and is also greater than the initial growth rate of bubbles growing on a pulsating surface. However, $dr/d\theta$ for both the surfaces practically remain approximately the same in the second stage ($\theta = 0.2 \times 10^{-3}$ to 1.5×10^{-3} sec.). At this point most of the bubbles in the case of a vibrating wire would break off the surface, whereas for stationary wires they would still remain attached to the surface and continue to grow, but at a very slow rate. There is no third stage for bubbles on a pulsating surface. Thus the overall time a bubble needs to grow and leave the surface is much smaller for vibrating wires. However, as may be expected, the diameter at break-off is smaller. The maximum value for the generating period of a bubble growing on the pulsating wire is 2.5 msec., whereas for a stationary wire it is often three to four times this value. The values of bubble radius for times greater than 3 msec. are not plotted in Figure 5.

WAITING PERIOD

The waiting period was studied on the same individual natural active site where it varied from 0.2 to 18 msec. in a random fashion. The waiting period was usually

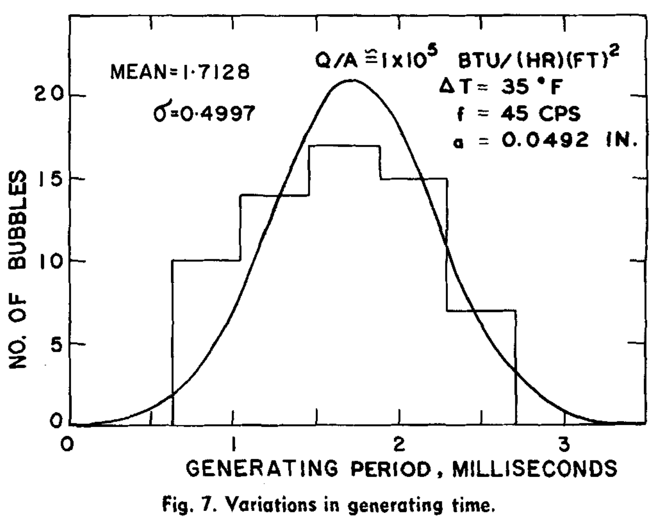

Fig. 7. Variations in generating time.

* Experimental data have been deposited as document 9435 with the American Documentation Institute, Photoduplication Service, Library of Congress, Washington 25, D.C., and may be obtained for $1.25 for photoprints or 35-mm. microfilm.

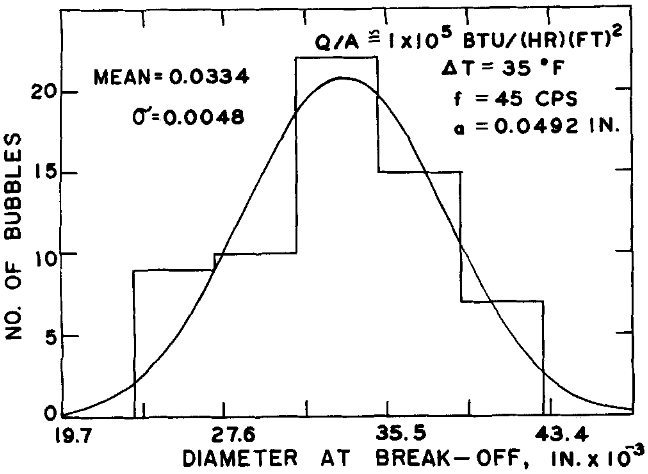

Fig. 8. Variations in diameter at break-off.

much longer than the generating period. It appears that oscillations of the heat transfer surface introduce extra disturbance and it takes much longer for the thermal boundary layer to develop so as to initiate bubble growth on the potential active site. On the other hand, a comparatively shorter and more uniform waiting period was observed for a stationary wire. Figure 6 shows the waiting period for bubbles which are generated in succession on the same nucleation site of an oscillating heat transfer surface.

GENERATING PERIOD AND DIAMETER AT BREAK-OFF

As predicted by Zuber *(10)* and others, there is an inflection point for the bubble diameter at breakoff when the bubble population density attains a value such that the interfering action of one bubble overlaps the interfering action of the neighboring bubble, and hence the bubble experiences an additional pressure to leave the surface. It may be expected that bubbles would experience the similar situation for a vibrating wire, even though the bubble population density has not reached a value as mentioned above. The equations such as proposed by Fritz *(11)* will not be valid in estimating the diameter at break-off. From the film analysis it was found that the diameters at break-off and their generating periods follow normal patterns of distribution. Theoretical normal curves were fitted to these properties as shown in Figures 7 and 8. The normal distribution curves could be represented by

$$N = \frac{1}{0.4997\sqrt{2\pi}}\, e^{-1/2}\left(\frac{\theta_g \times 10^{-3} - 1.7123}{0.4997}\right)^2$$

for generating time and

$$N = \frac{1}{0.0048\sqrt{2\pi}}\, e^{-1/2}\left(\frac{d - 0.0334}{0.0048}\right)^2$$

for diameter at break-off. The frequency of bubble emission defined as $1/(\theta_w + \theta_g)$ was found to be slightly higher for a vibrating wire.

CONCLUSIONS

Vibrations of the heat transfer surface influence the saturated nucleate boiling of water. An increase in frequency or amplitude increases the heat flux. It is postulated that the imposed vibrations result in a decrease in effective thickness of the boundary layer and a flatter temperature profile across it. The generating period for bubbles and their diameters at break-off are less for a vibrating wire. The distribution of these variables can be represented by normal distribution curves. The waiting period is longer and more random in nature as compared to stationary surfaces. A slight increase in bubble emission frequency is noticed for a pulsating wire.

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NOTATION

- a = total displacement of amplitude of vibration of wire, in.
- c = constant
- D = diameter of platinum heating wire, in.
- d = diameter of bubble at break-off, in.
- f = frequency of vibration of the wire, cycles/sec.
- h = heat transfer coefficient at bubble wall, B.t.u./(hr.) (sq.ft.) (°F.)
- N = number of bubbles
- n = constant
- Q/A = heat flux from test section to boiling water, B.t.u./(hr.) (sq.ft.)
- r = mean radius of the bubble, in.
- T = temperature, °F.
- v = average wire velocity $2af/12$, ft./sec.

Greek Letters

- ΔT = temperature difference between the wire and water, °F.
- θ = time, sec.
- σ = standard deviation of the distribution

Subscripts

- g = generating period a bubble needs to grow and leave the surface
- S = saturation condition
- W = wire
- w = waiting period

LITERATURE CITED

1. Gunther, F. C., and F. Kreith, "Heat Transfer and Fluid Mechanics Institute", pp. 113-121, Berkeley, Calif. (1949).
2. Rosenhow, W. M., and J. A. Clark, *Trans. Am. Soc. Mech. Engrs.*, **73**, 609 (1951).
3. Strenge, P. H., Aluf Orell, and J. W. Westwater, *AIChE J.*, **7**, 578 (1961).
4. Schweppe, J. L., and A. S. Foust, *Chem. Eng. Progr. Symp. Ser. No. 5*, **49**, 77 (1953).
5. Robinson, D. B., and D. L. Katz, *Chem. Eng. Progr.*, **47**, 317 (1951).
6. Lemlich, Robert, *Ind. Eng. Chem.*, **47**, 1175 (1955).
7. Gaertner, R. F., and J. W. Westwater, *Chem. Eng. Progr. Symp. Ser. No. 30*, **56**, 39 (1960).
8. Plesset, M. S., and S. A. Zwick, *AIChE J.*, **25**, 493 (1954).
9. Forster, H. K., and Novak Zuber, *J. Appl. Phys.*, **25**, 474 (1954).
10. Zuber, Novak, *AECU 4439*, AEC (1959).
11. Jakob Max, "Heat Transfer," Vol. 1, Wiley, New York, p. 614 (1949).
12. Burgess, G. K., and H. LeChatelier, "The Measurement of High Temperatures," 3 ed., pp. 194-237, 493-495, Wiley, New York (1912).

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